

Truncating Cross-Sectional Groundwater Models under Wetlands

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Abstract: Cross-sectional models, which represent two-dimensional flow in the vertical plane, tend to have problematic aspect ratios since the aquifer thickness is often small compared to the lateral extent of the flow domain. For that reason, the model domain is usually limited to the immediate area of interest, for instance the aquifer section underneath a dam. We propose a Cauchy boundary condition to represent flow from remote wetlands that are left out of the truncated model. The resistance to flow inherent to such a boundary depends on the aquifer properties and the resistance to flow through the wetland bottom. While the Cauchy boundary condition is based on the Dupuit-Forchheimer approximation to flow underneath the remote wetlands, the error appears to be negligible (less than 0.6%) for most practical cases, including flow in stratified aquifers. For the case of multiple aquifers underneath the wetlands, the total flow in the truncated model can be a few percent in error, which is typically acceptable for most engineering applications. The approach is illustrated with an application near a levee-borrow canal setting in the Florida Everglades.

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Introduction

Cross-sectional models of groundwater flow represent two-dimensional groundwater flow in the vertical plane and may offer an efficient analysis of flow underneath and near hydraulic structures (e.g., Harr 1962), flow toward canals (e.g., Chahar 2007), or for modeling saltwater intrusion in coastal aquifers (e.g., Bear and Verruijt 1987). In the Florida Everglades region wetland and groundwater flow interactions have been studied using cross-sectional models near levees and associated borrow canals (Wilsnack and Kelson 2007). The challenge is to include leakage from wetlands on either side of the levee and canal without extending the model underneath the entire wetland domain, which would lead to problematic model aspect ratios and associated computational difficulties. We illustrate this problem, and our solution to it, for the more general problem of a levee-borrow canal combination with a wetland on one side and a remote surface water boundary (e.g., another canal) on the other side (see Fig. 1). The figure depicts a cross section of a levee with borrow canal and underlying aquifer. Far to the left of the canal is a Dirichlet boundary condition (another canal not shown in Fig. 1) while to the right of the levee is a wetland area of which only a small part near the levee is shown in the figure. Extending the model to

include all of the wetlands to the right of the levee and all of the aquifer, up to the remote canal on the left, is impractical. The resulting aspect ratio of the model domain would become problematic for numerical models. For instance, in the event of a finite difference model, grid design problems would ensue, resulting in very thin and elongated grid cells which tend to cause numerical difficulties (Anderson and Woessner 1992). Similarly, in analytic element models line sinks and line doublets would have collocation points that are very close together, resulting in a poorly conditioned problem (Haitjema 1995). The fact that the lateral extent of aquifers is usually one or more orders of magnitude larger than the aquifer thickness also results in rather uninteresting flow patterns: almost exclusively horizontal flow throughout most of the flow domain.

To keep the model aspect ratio manageable the model domain is limited to an area similar as depicted in Fig. 1 thereby leaving the remote Dirichlet boundary (canal) to the left and much of the wetlands to the right out of the model. This can only be done if the flow due to these remote boundaries is accounted for in the model at least in some approximate manner. For the case of the remote canal to the left of the domain in Fig. 1 it is customary to introduce a Cauchy boundary condition also referred to as a “general head boundary” or “head-dependent flux boundary” (see Anderson and Woessner 1992). The wetland boundary condition to the right of Fig. 1, however, is more complicated. The wetland will leak water into the aquifer over an a priori unknown distance from the levee and at a rate that varies with the distance from the levee. In this paper we propose to represent this remote wetland boundary condition with a similar Cauchy boundary condition as used to include the effect of the remote canal to the left of the model domain in Fig. 1 but with a resistance parameter that is calculated differently. This Cauchy boundary can also be used if the wetlands are replaced by a lake with bottom sediments that form a resistance layer. Additionally, while the use of this Cauchy boundary is most important to cross-sectional models, it may also be used in two-dimensional flow models in the hori-

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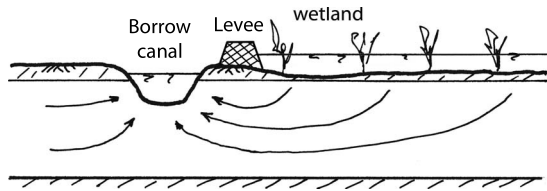


Fig. 1. Cross section over an aquifer underneath a levee with borrow canal and an adjacent wetland

zontal plane, for which general head boundaries are routinely used.

The paper is organized as follows. First we introduce the proposed Cauchy boundary. Next we validate the underlying assumptions by comparison with exact solutions. We also assess the performance of the truncated model for stratified aquifers and multiaquifer settings. Finally we present an application of the truncated modeling approach in the Florida Everglades wetlands.

Including Remote Boundaries in the Truncated Model

Assume that in Fig. 1 there is a canal at a distance L to the left of the model domain (the domain shown in the figure). We can incorporate that remote Dirichlet boundary using a Cauchy type boundary condition, which represents the resistance to flow in the aquifer section to the left of the model domain. Using Darcy's law for the flux q_x in the x -direction in the remote aquifer domain we write

$$q_x = k \frac{\phi_c - \phi_l}{L} = \frac{\phi_c - \phi_l}{c_D} \quad (1)$$

where k = aquifer hydraulic conductivity; ϕ_c = head at the canal at a distance L from the left-hand model domain boundary; and ϕ_l = a priori unknown head at the left-hand model domain boundary. The resistance c_D of the Cauchy boundary follows from Eq. (1) as

$$c_D = \frac{L}{k} \quad (2)$$

This approach is routinely used by MODFLOW modelers employing a general head boundary or a head-dependent flux boundary albeit by specifying a "conductance" [which is the inverse of Eq. (2)] and the head ϕ_c (Anderson and Woessner 1992; McDonald and Harbaugh 1988). In analytic element models the boundary will be represented by line sinks with a specified head ϕ_c and resistance c_D .

We can use a similar Cauchy type boundary condition at the right-hand side of the model domain in Fig. 1 to represent flow from the remote wetlands outside the model domain. We will explain the approach by use of Fig. 2. The upper sketch in Fig. 2 depicts a vertical section of an aquifer underneath a wetland that extends (infinitely) far to the right [Fig. 2(a)]. The wetlands seep water into the underlying aquifer which discharges into the canal on the left. Below it [Fig. 2(b)], the domain is shown that will be included in the cross-sectional model, whereby the flow due to remote wetlands to the right of the model domain is approximated by flow from a Cauchy type boundary with resistance c_b . The left-hand boundary at $x=0$ is a constant head boundary with a known head ϕ_0 . The water elevation in the wetland is ϕ_w . The wetland is separated from the aquifer by a sediment layer of

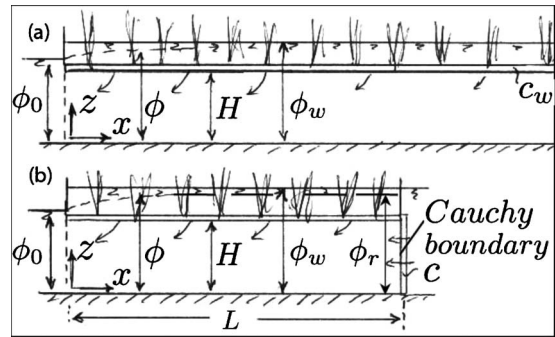


Fig. 2. Aquifer with overlying wetland. (a) Wetland extends infinitely far to the right. (b) Model domain has a Cauchy type boundary to represent flow from the wetlands to the right that are not included in the model domain.

thickness d and vertical hydraulic conductivity k_s , which results in a resistance to flow $c_w = d/k_s$ days. The origin of a Cartesian x, z -coordinate system is located at the aquifer bottom and at the left-hand constant head boundary. If the leakage from the wetland into the aquifer occurs over a sufficiently large portion of the wetland, resistance to horizontal flow dominates and the Dupuit-Forchheimer approximation may be adopted, ignoring any resistance to vertical flow inside the aquifer. For that case, the discharge component Q_x in the aquifer becomes [see Verruijt (1970), p. 30, Eq. (4.10)]

$$Q_x = -kH \frac{\phi_w - \phi_0}{\lambda} e^{-x/\lambda} \quad (3)$$

where the leakage factor λ is referred to here as the *characteristic leakage length* and defined as (Verruijt 1970)

$$\lambda = \sqrt{kHc_w} \quad (4)$$

We will determine the appropriate choice for the resistance c_b for the Cauchy boundary in Fig. 2(b) by comparing the flow at that boundary with the flow coming out of the wetlands that are located outside the model domain. Assuming that the wetlands extend infinitely far to the right of Fig. 2(b) the total flow out of the remote wetlands that enters the model domain through the right-hand boundary can be obtained from Eq. (3) by setting $x=0$ and replacing ϕ_0 by the a priori unknown head ϕ_r at the right-hand boundary of the model domain. Thus

$$Q = -kH \frac{\phi_w - \phi_r}{\lambda} \quad (5)$$

Dividing Eq. (5) by the aquifer thickness H we obtain the flux q_x (m/day) at the right-hand model domain boundary

$$q_x = -k \frac{\phi_w - \phi_r}{\lambda} \quad (6)$$

which can be compared to flow in a fictitious confined aquifer with Dirichlet boundary conditions

$$q_x = -\frac{\phi_w - \phi_r}{c_b} \quad (7)$$

The minus signs in Eqs. (6) and (7) reflect the fact that the flow is from the right to the left, thus in negative x -direction. Comparing Eqs. (6) and (7) yields a resistance c_b on the right-hand domain boundary of

$$c_b = \frac{\lambda}{k} \quad (8)$$

When defining the right-hand boundary with head-dependent flux cells in MODFLOW (or line sinks in an analytic element model) the head is set at the wetland water surface elevation ϕ_w . Note that the properties of the right-hand Cauchy boundary do not depend on where the model is truncated, hence what the size L of the model domain is, which is unlike the Cauchy boundary for a remote Dirichlet boundary [see Eq. (2)].

Validation

There are two approximations involved with the newly introduced Cauchy boundary that require some justification. First, in writing Eq. (3) the Dupuit-Forchheimer approximation was applied. Second, because of that, the application of the Cauchy boundary involves a resistance and a head that are constant over the aquifer height, which is an approximate boundary condition to the two-dimensional flow problem in the model domain [Fig. 2(b)]. We will test these two approximations separately.

The exact solution to two-dimensional flow in Fig. 2(a) has been adapted from Bruggeman (1999), problem 355.13 on p. 299, and leads to the following head $\phi(x, z)$ inside the aquifer:

$$\phi(x, z) = \phi_w - 2(\phi_w - \phi_0) \sum_{n=0}^{\infty} \frac{e^{-(\alpha_n/H)x} \sin(\alpha_n)}{\alpha_n \left(1 + \frac{\varepsilon}{\alpha_n^2 + \varepsilon^2}\right)} \cos\left(\frac{\alpha_n z}{H}\right) \quad (9)$$

where

$$\varepsilon = \frac{H}{kc_w} \quad (10)$$

and where α_n = roots of $\alpha \tan(\alpha) = \varepsilon$ ($n=0, 1, 2, \dots, \infty$). For α_0 the root is in the first quadrant ($0 < \alpha < \pi/2$), for α_1 the root is in the third quadrant ($\pi < \alpha < 3\pi/2$), etc. The total flow integrated over the aquifer height at some point x from the left-hand boundary follows from

$$Q_x = \int_0^H q_x dz = \int_0^H -k \frac{\partial \phi}{\partial x} dz \quad (11)$$

which yields with Eq. (9)

$$\phi(x, z) = \phi_w - 2(\phi_w - \phi_0) \sum_{n=0}^{\infty} \frac{\sin(\alpha_n) \cos\left(\frac{\alpha_n}{H} z\right) \left(1 - \frac{\varepsilon'}{\alpha_n}\right) e^{-(\alpha_n/H)(L-x)} + \left(1 + \frac{\varepsilon'}{\alpha_n}\right) e^{(\alpha_n/H)(L-x)}}{\alpha_n \left(1 + \frac{\varepsilon}{\alpha_n^2 + \varepsilon^2}\right) \left(1 - \frac{\varepsilon'}{\alpha_n}\right) e^{-(\alpha_n/H)L} + \left(1 + \frac{\varepsilon'}{\alpha_n}\right) e^{(\alpha_n/H)L}} \quad (14)$$

where $\alpha_n \tan \alpha_n = \varepsilon$ ($n=0, 1, 2, \dots, \infty$) and ε is defined by Eq. (10).

The parameter ε' is defined as

$$\varepsilon' = \frac{H}{kc_b} = \sqrt{\varepsilon} \quad (15)$$

$$Q_x = -2k(\phi_w - \phi_0) \sum_{n=0}^{\infty} \frac{e^{-(\alpha_n/H)x} \sin^2(\alpha_n)}{\alpha_n \left(1 + \frac{\varepsilon}{\alpha_n^2 + \varepsilon^2}\right)} \quad (12)$$

To assess the adequacy of the Dupuit-Forchheimer approximation, the total outflow from the wetlands, and thus from the aquifer at $x=0$, is evaluated with both Eqs. (3) and (12) for various values of λ/H and the resulting flows are compared. For $\lambda/H = 1, 1.5, 2,$ and 2.5 we find that the Dupuit-Forchheimer outflows are 26, 10, 3.3, and 2.2% larger than the exact outflow, respectively. Therefore, we propose to accept the Dupuit-Forchheimer approximation for all cases where

$$\lambda \geq 2H \quad (13)$$

Assuming that Eq. (13) is satisfied, we can indeed represent the flow in the aquifer underneath the remote wetlands, which occur outside the model domain shown in Fig. 2(b), as the Dupuit-Forchheimer flow.

The limitation on λ as given by Eq. (13) is of little practical concern as may be seen as follows. The characteristic leakage length λ is a measure of the spatial distribution of the leakage. For instance, with reference to Eq. (3), we compare the aquifer discharge Q_x at a distance of 3λ from the left-hand boundary ($x=3\lambda$) with that of the total discharge from the wetland, which is Q_x at $x=0$, and find that 95% of the flow occurs over this distance 3λ (see also Hunt et al. 2003). For values of $\lambda < 2H$ the model domain in Fig. 2(b) can easily be extended to 3λ without leading to a problematic aspect ratio (e.g., $L=6H$ if $\lambda=2H$). Such a model includes 95% of the leakage from the wetlands so that any error in flow due to the Cauchy boundary is an error in only 5% of the total flow, hence rather inconsequential.

The use of the Cauchy boundary on the right-hand side of the model domain [Fig. 2(b)] is approximate since the application of the Dupuit-Forchheimer approximation outside the model domain implies that the head is constant over the aquifer height along the Cauchy boundary. To assess the error resulting from this approximation we compared an exact solution to the flow problem depicted in Fig. 2(b), which includes the Cauchy boundary, with the exact solution [Eq. (12)] for the problem depicted in Fig. 2(a), which includes the remote wetlands explicitly. The former solution has been obtained by inspection of the solutions to problems 355.13 and 355.15 in Bruggeman (1999) (pp. 299 and 300). The solution for the head $\phi(x, z)$ is given here without derivation

where use has been made of Eqs. (8) and (4). Solution (14) for $\phi(x, z)$ may be shown to satisfy the governing differential equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (16)$$

and the boundary conditions

$$x=0; \phi = \phi_0 \quad (17)$$

$$x=L; k \frac{\partial \phi}{\partial x} = \frac{\phi_w - \phi}{c_b} \quad (18)$$

$$z=0; \frac{\partial \phi}{\partial z} = 0 \quad (19)$$

$$z=H; q_z = \frac{\phi - \phi_w}{c_w} = -k \frac{\partial \phi}{\partial z} \quad (20)$$

Following the same procedure as in Eq. (11) we obtain the total flow in the truncated aquifer in Fig. 2(b) as

$$Q_x = -2k(\phi_w - \phi_0) \sum_{n=0}^{\infty} \frac{\sin^2(\alpha_n)}{\alpha_n \left(1 + \frac{\varepsilon}{\alpha_n^2 + \varepsilon^2}\right)} \frac{\left(1 - \frac{\varepsilon'}{\alpha_n}\right) e^{-(\alpha_n H)(L-x)} - \left(1 + \frac{\varepsilon'}{\alpha_n}\right) e^{(\alpha_n H)(L-x)}}{\left(1 - \frac{\varepsilon'}{\alpha_n}\right) e^{-(\alpha_n H)L} + \left(1 + \frac{\varepsilon'}{\alpha_n}\right) e^{(\alpha_n H)L}} \quad (21)$$

We evaluated Q_x at $x=0$ in Fig. 2(b) by applying Eq. (21) with different values of L/H (different model domain sizes) and with different values of λ/H . These results were compared to Q_x in Fig. 2(a) as obtained from Eq. (12). A percent error relative to the value obtained from Eq. (12) is plotted in Fig. 3. In most practical cases the aspect ratio of the model domain will not be smaller than $L/H=2$ which, with reference to Fig. 3, keeps the error less or equal to 0.6%, depending on the value of λ .

Verifications Using the Analytic Element Method

To further test the applicability of the Cauchy boundary for representing flow in or out of the model domain due to remote wetlands we developed three different analytic element models using GFLOW (Haitjema 1995). The first GFLOW model is a representation of the flow domain depicted in Fig. 2(b) whereby the GFLOW solution is compared to the closed form analytic solution (21). The purpose of this exercise is to assess the suitability of GFLOW in solving these two-dimensional flow problems. The left-hand constant head boundary is represented by a string of 30 line sinks without resistance, the right-hand Cauchy boundary by a string of 31 line sinks with resistance, and the horizontal wet-

land bottom (also a Cauchy type boundary condition) by a string of 67 line sinks with resistance. The model domain has been surrounded by a closed horizontal barrier (modeled with line doublets) that forms a no-flow boundary. For further reading on the application of analytic element models the reader is referred to Haitjema (1995). We tested the case for $L/H=3$ with $L=30$ m and $H=10$ m. The hydraulic conductivity of the aquifer is $K=100$ m/day and the resistance of the wetland bottom is $c_w=1$ day. The head at the left-hand boundary is $\phi_0=11$ m, while the water level in the wetland is $\phi_w=21$ m, both measured with respect to the aquifer bottom. A set of GFLOW-generated equipotentials and streamlines for the flow domain in Fig. 2(b) is reproduced in Fig. 4(a). The exact outflow rate at the left-hand boundary, as calculated by Eq. (21), equals $Q_x=-311.729$ m²/day, while the flow on the left-hand boundary in the GFLOW model was found to be $Q_x=-312.19$ m²/day, which differs less than 0.15% from the exact solution. Based on these results we proceeded to use GFLOW to assess the performance of the Cauchy boundary for the cases of a stratified aquifer and multiple aquifers underneath the wetlands for which we have no exact analytic solutions at hand.

Stratified Aquifer

In Fig. 4(b) GFLOW-generated equipotentials and streamlines are shown for the case where the aquifer in Fig. 2(b) is subdivided into three layers. The upper and lower layers have a thickness of 3.5 m and a hydraulic conductivity of 100 m/day, while the layer in the middle is 3 m thick and has a conductivity of 20 m/day. The contrasts in hydraulic conductivity are modeled by use of line doublets (Haitjema 1995). For modest contrasts in hydraulic conductivity the stratified aquifer can be seen as a single Dupuit-Forchheimer aquifer and the characteristic leakage length λ is calculated using the total transmissivity T

$$\lambda = \sqrt{T c_w} \quad (22)$$

whereby T is the sum of the transmissivities of the three layers. The resistance for the Cauchy boundary on the right-hand side differs for the three layers and is calculated as

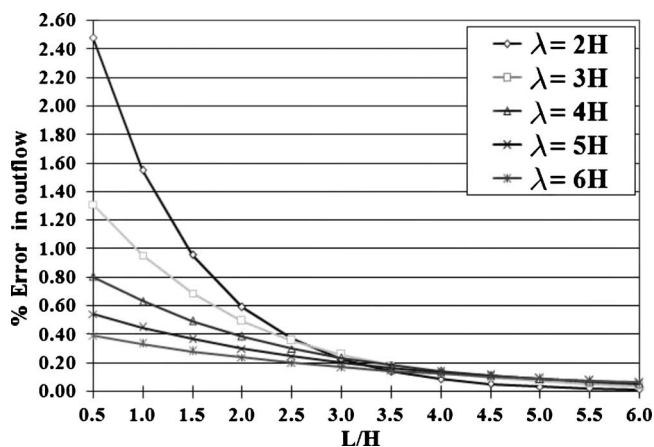


Fig. 3. Percent error in total outflow from the truncated model as compared to the infinitely long model

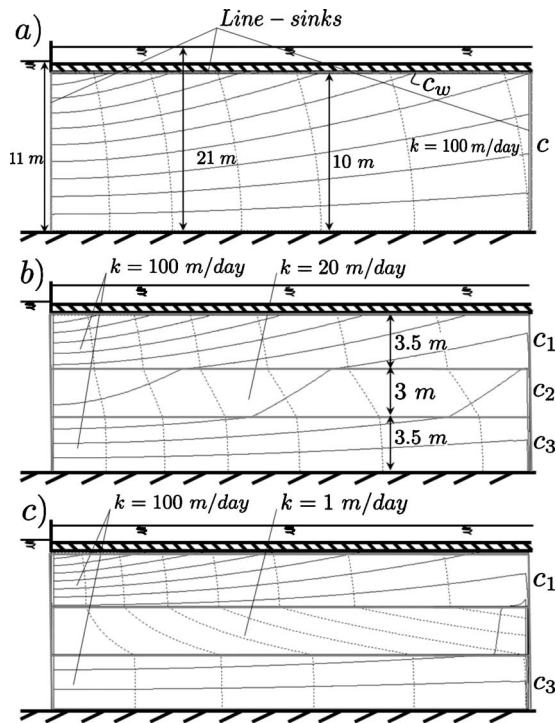


Fig. 4. Equipotentials (vertical dotted curves) and streamlines (horizontal solid curves) for the case of (a) homogeneous aquifer, (b) stratified aquifer, and (c) two aquifers separated by an aquitard. Each of the three figures represents the flow domain depicted in Fig. 2(b). The left-hand boundary is an equipotential with a head of 11 m, while subsequent equipotentials increase in head by 1 m. Flow between each pair of streamlines is 0.1 of the outflow out of the aquifer as reported in the text. Note that the figures are not to scale with the vertical dimension of the aquifer exaggerated for clarity.

$$c_i = \frac{\lambda}{k_i} \quad (i = 1, 2, 3) \quad (23)$$

where c_i = resistance of the Cauchy boundary in layer i and k_i = hydraulic conductivity of layer i . Eq. (23) is the result of applying the analysis represented by Eqs. (5)–(8) for each layer, thereby realizing that the head ϕ_r is the same for each layer, a consequence of the Dupuit-Forchheimer approximation. For the case presented in Fig. 4(b), the numerical values are $\lambda = 27.5681$ m, $c_1 = c_3 = 27.5681/100 = 0.275681$ days, and $c_2 = 27.5681/20 = 1.3784$ days whereby the layers in Fig. 4(b) are numbered from top to bottom. The total outflow on the left-hand side of the aquifer in Fig. 4(b) is 270.465 m³/day. Since we do not have an exact solution to the case in Fig. 4(b) we also constructed a GFLOW model of 180 m length, which is more than 6.5λ (thus including nearly 100% of the wetland leakage). The flow coming into the right-hand side of that model (from the Cauchy boundary) is less than 0.5 m³/day, hence any error in that small amount of flow is insignificant when compared to the total outflow on the left-hand side of over 270 m³/day. The total outflow of the long model is 269.567 m³/day. Considering the latter as the “true” outflow, the error due to the truncation of the model domain is 0.33% .

Multiple Aquifers

In Fig. 4(b), the contrast in hydraulic conductivity between the layers is modest, a factor 5, which makes the Dupuit-Forchheimer

approximation still acceptable provided Eq. (13) is satisfied. In Fig. 4(c), however, the hydraulic conductivity of the center layer has been lowered to 1 m/day, two orders of magnitude below the conductivity of the other two layers, which changes the conceptual model into that of a two-aquifer system separated by an aquitard. In this case we assume each layer to be isotropic. We tested the Cauchy boundary approximation for the truncated model domain in Fig. 4(c) while ignoring any (horizontal) inflow through the center layer (the aquitard) from the remote aquifer to the right of the model domain. The resistances of the Cauchy boundaries inside the upper and lower aquifers were calculated as follows. The resistance c_1 for the upper Cauchy boundary is

$$c_1 = \frac{\lambda_1}{k_1} \quad (24)$$

with

$$\lambda_1 = \sqrt{T_1 c_w} \quad (25)$$

The resistance c_3 of the lower Cauchy boundary is

$$c_3 = \frac{\lambda_3}{k_3} \quad (26)$$

with

$$\lambda_3 = \sqrt{T_3(c_w + H_2/k_2)} \quad (27)$$

The indices refer to the layers in Fig. 4(c), which are numbered from top to bottom. It is seen from Eqs. (25) and (27) that each of the two “aquifers” has its own λ parameter and that the resistance used to calculate the characteristic leakage length for the lower aquifer includes the resistance due to the separating aquitard (Layer 2) [see Eq. (27)]. Note that we ignored the resistance H_1/k_1 for Layer 1 on account of the relatively large value of k_1 . For the case presented in Fig. 4(c) the numerical values are $\lambda_1 = 18.708$ m, $c_1 = 18.708/100 = 0.18708$ days, $\lambda_3 = 37.4165$ m, and $c_3 = 37.4165/100 = 0.374165$ days. The total outflow on the left-hand side of the aquifer in Fig. 4(c) is 244.9026 m³/day. We also constructed a GFLOW model of 180 m length, which is about $9.6\lambda_1$ or $4.8\lambda_3$. The total outflow of the long model is 238.9723 m³/day. Again, considering the latter the true outflow the error due to the truncation of the model domain is now 2.5% .

Application to the Florida Everglades

We illustrate the use of the new Cauchy boundary by briefly summarizing an application near a levee (L-67A) and its borrow canal in the Florida Everglades. The L-67A levee is located within the remnant Everglades in southeastern Fla., approximately 30-km northwest of the Miami Metropolitan area (see Fig. 5). Regional surface water impoundments (wetlands) denoted as Water Conservation Areas 3A (WCA3A) and 3B (WCA3B) lie northwest and southeast of L-67A, respectively. The area consists of subtropical wetlands underlain by peat, marl, and freshwater limestone (Fish and Stewart 1991). The aim of our study is to assess the amount of leakage from WCA3A into WCA3B along with its distribution over the various aquifer strata below the levee. This study is similar to one conducted earlier by Wilsnack and Kelson (2007) for levee L-31N located south of the current study area (see Fig. 5). Since the wetland areas extend far from the levee and its borrow canal, our truncated modeling approach is essential to arrive at a functional aspect ratio of our model domain.

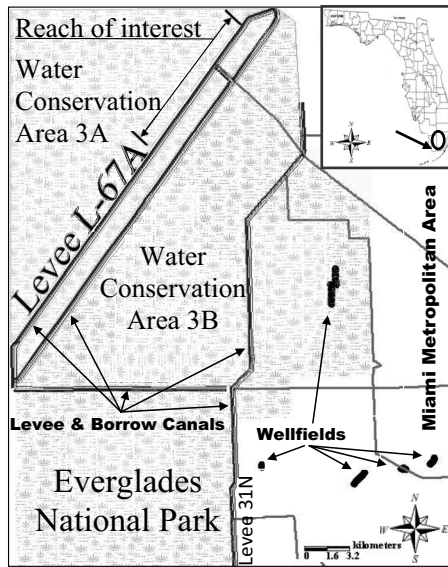


Fig. 5. Location of levee L-67A near Miami

Hydrogeology and Conceptual Model

The position of levee L-67A, its borrow canal, and underlying geological strata are depicted in Fig. 6. The situation in Fig. 6 differs from the one in Fig. 1 in that the canal is submerged and wetlands occur on both sides of the levee with groundwater flow from left to right. The hydraulic conductivity and thicknesses of the various hydrogeologic zones below L-67A are specified in Table 1 (Columns 3 and 4). For a more detailed discussion on the hydrogeology in this area the reader is referred to Fish and Stewart (1991). The zones below L-67A have differing hydraulic conductivities with contrasts between two and four orders of magnitude so that the characteristic leakage length λ for each layer has been calculated using a multiaquifer approach [see Eqs. (24)–(27)]. This time, however, the resistances of all layers (aquifers as well as aquitards) have been included, which result in [compare Eq. (27)]

$$\lambda_i = \sqrt{k_i H_i \sum_{j=1}^{i-1} c_j} \quad (28)$$

where c_j follows from

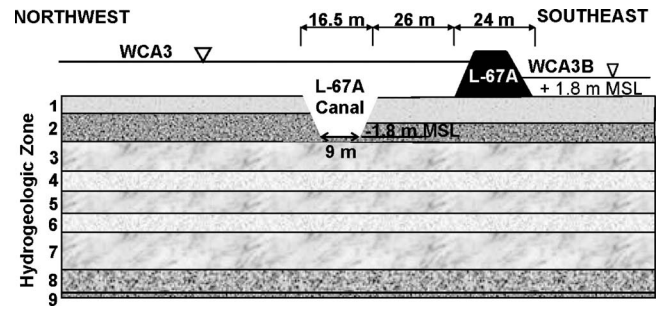


Fig. 6. Conceptualization of levee L-67A with borrow canal and underlying hydrogeology

$$c_j = \frac{H_j}{k_j} \quad (29)$$

with i and j in Eqs. (28) and (29) denoting the zone numbers in Table 1. All zones are assumed to have an isotropic conductivity; thus the values for k reported in Table 1 reflect both the horizontal and vertical hydraulic conductivities. The resulting resistances and λ values are reported in Columns 5 and 6 of Table 1, respectively. Finally, the resistance $(c_b)_i$ for each of the Cauchy boundaries in layer i (both on the left- and right-hand sides of the model domain) follows from

$$(c_b)_i = \frac{\lambda_i}{k_i} \quad (30)$$

and is reported in Column 7 of Table 1.

The groundwater flow problem has been solved with the analytic element model GFLOW for the scenario where the surface water stage in WCA-3A is 0.61 m higher than the stage in WCA-3B. The model domain extends 600 m to the left in Fig. 6, under WCA3A, and 540 m to the right under WCA3B. These distances translate into 1.4 and 1.2 times the λ value for Zone 7, respectively, the largest λ value in the system (see Table 1). Zone 1 with the peat, muck, and marl was considered a wetland bottom layer and included as such in the model as a line sink with resistance, hence no horizontal flow was calculated in that zone. The bottom of Zone 9 was considered a no-flow boundary. Furthermore, the model was assigned a default hydraulic conductivity value of 0.3 m/day while any layer with a different hydraulic conductivity was modeled as an inhomogeneity whose boundaries are comprised of a series of line doublets. The canal sidewalls were designated as line sinks with no resistance (Chin 1990; Merritt 1995) and with the same stage as the adjacent wetland. The resulting flows Q for

Table 1. Zone Properties and Flows

Zone	Description	k (m/day)	H (m)	c (day)	λ (m)	c_b (day)	Q (m ² /day)
1	Peat, muck, and marl	0.3	0.91	3.03	—	—	—
2	Dense limestone and shells	0.3	3.96	13.2	1.90	6.24	1.23×10^{-3}
3	Porous limestone and shells	3,048	3.05	0.0010	385.45	0.13	4.77
4	Limestone and shells	96	1.52	0.0158	48.46	0.50	0.08
5	Porous limestone and shells	3,048	1.83	0.0006	298.80	0.10	2.68
6	Limestone and shells	96	1.52	0.0158	48.49	0.50	0.073
7	Porous limestone and shells	3,048	3.66	0.0012	422.78	0.14	5.76
8	Sandstone and shells	9.6	2.13	0.2212	18.15	1.88	0.010
9	Dense limestone and shells	0.3	0.91	3.03	2.13	6.98	1.73×10^{-4}

each of the zones underneath the levee are reported in Column 8 of Table 1. These flows are reported in terms of square meter per day as they represent cubic meters per day per meter of aquifer perpendicular to the plane of Fig. 6. Note that while Layers 2 and 9 were treated as aquifers, their low transmissivities resulted in almost negligible flow rates, hence alternatively they might have been treated as part of the wetland bottom and aquifer base, respectively. It was found that almost half (47.7%) of the total seepage out of WCA3A comes from outside the model area and enters the model across the Cauchy boundaries on the left-hand side of the model domain. Similarly, 54.8% of the flow entering WCA3B exits through the Cauchy boundaries on the right-hand side of the model domain.

In this case the truncated model domain extends less than 1.4 times the largest λ value (found in Layer 7; see Table 1) on either side of the borrow canal or levee, which already stretches the feasible aspect ratio of the model (19.49 m high by 1140 m wide). This is why we omitted an equipotential and streamline plot for this case. Without our approach to truncating the model area one would have liked to capture at least 95% of the leakage from these wetlands, hence extending the model domain to at least 3λ on either side of the levee (Hunt et al. 2003). This would have created a problematic (from a numerical perspective) aspect ratio of 19.49 m high by 2,537 m wide or wider.

Conclusions

Cross-sectional models are often much longer than their height. This awkward aspect ratio makes for difficult model setup and can even lead to solution instabilities or inaccuracies. The model domain may be truncated at some distance from the area of interest, provided that the flow from the remote aquifer zones that are not included in the model is properly accounted for in the truncated model domain. The remote head-specified boundaries can be and routinely are simulated by means of a general head boundary, which is a Cauchy type boundary that includes the resistance to horizontal flow in the aquifer zone that is outside the model domain. We propose a new Cauchy type boundary to include remote wetlands (or a lake) with a bottom resistance c_w . For this case, the Cauchy boundary has a resistance $c = \lambda/k$. The parameter λ is the *characteristic leakage length* defined as $\lambda = \sqrt{kHc_w}$, where H is the aquifer thickness. This new Cauchy boundary is adequate as long as the flow regime underneath the remote wetlands it represents can be approximated as the Dupuit-Forchheimer flow. The error associated with this approximation appears very small for most practical cases, in the order of 0.6% or less.

The approach can be extended to include aquifer stratification and can even be applied to cases of multiaquifer flow. For the latter case, however, the approximation yields somewhat less accurate results, 2.5% error in total flow for a test case presented in

this paper. The impact of the approximation on the accuracy of the flow in the model domain depends on the (remaining) width of the truncated model. For instance, if the wetland's portion inside the truncated model domain is 3λ in length it already provides 95% of all wetland leakage (Hunt et al. 2003). For such a case, any error due to the truncation process is much less than 5%; in fact, much less than the 0.6% error found in our validations. Our new Cauchy boundary has been applied to cross-sectional models in the Florida Everglades as illustrated in this paper by the modeling of flow near and underneath the L-67A levee and borrow canal. We showed that our truncated modeling approach was important to maintain a workable aspect ratio of the model domain.

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