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Stochastic description of water table fluctuations in wetlands

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[1] Wetlands are crucial ecosystems which provide several functions, beneficial both to human beings and to the environment. Despite such importance, quantitative approaches to many aspects of wetlands are far from being adequate, above all the interaction between rainfall, vegetation, soil moisture and groundwater depth. Starting from a previously developed model for below-ground stochastic water level fluctuations, we extend it to consider the case of waterlogging. The extended model is now suitable for describing the long-term probability distribution of water table depth in temporarily inundated wetland sites, whose hydrologic input is dominated by stochastic rainfall. The extended model performs well when compared to real data collected in the Everglades National Park (Florida, US), confirming its capability to capture the stochastic variability of wetland ecosystems.
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1. Introduction

[2] Wetlands cover approximately 6%—about 8 million km^2—of the Earth's land area [*Mitsch and Gosselink*, 2000; *Reddy and DeLaune*, 2008]. They provide a suite of functions critical to human's livelihood, be they health-related, cultural, social, or ecological, including, but not limited to, being habitats of disease-carrying vectors (e.g., malaria-carrying mosquitoes), providing buffer zones against hurricanes, controlling sediment transport (which affects navigability of waterways), filtering nutrients and contaminants (preventing eutrophication and algal bloom problems), and a repository of great biological diversity. More recently, wetlands have also been recognized as a crucial carbon storage in the global climate change context [*Fishchlin et al.*, 2007]. Despite such importance, quantitative approaches to many aspects of wetlands are far from being adequate. Therefore, improving our quantitative understanding of wetlands is necessary to our ability to maintain, manage, and restore these invaluable environments.

[3] One commanding factor in many wetland processes, be they hydrological, ecological, or biogeochemical, is water level fluctuation. Standing water in wetlands controls, among other things, oxygen transport into the soil, thereby controlling the reduction-oxidation potential in this submerged environment. This in turn dictates the fate of biogeochemical processes (e.g., microbial activities), of the plants and ultimately of the overall ecosystem functions and services [e.g., *Jackson et al.*, 1991; *Marani et al.*, 2006].

[4] Water level fluctuations in wetlands are a result of various interacting processes, such as precipitation, surface inflows and outflows, groundwater inflows and outflows, and evapotranspiration, some of which are stochastic. Yet, the description of water level fluctuation in these systems does not usually account for such stochasticity. For example, in wetland literature, this is often discussed under the topic of "hydroperiod." The term has traditionally been defined as generally as a seasonal pattern of water level in a wetland [e.g., *Mitsch and Gosselink*, 2000] or more specifically as the number of days in which a wetland is inundated [e.g., *Kadlec and Knight*, 1996], none of which addresses the stochasticity. Recently, research efforts have been invested in this direction [see *Rodriguez-Iturbe et al.*, 2007; *Daly et al.*, 2009]. In particular, *Laio et al.* [2009] and *Tamea et al.* [2009] quantitatively modeled the coupled stochastic dynamics of water table and soil moisture in groundwater-dependent ecosystems without considering waterlogging. A key feature in the dynamics of water table not accounted for in the aforementioned references is the probabilistic structure of water level fluctuations above the soil surface. To this purpose, we extend in this paper the stochastic model by *Laio et al.* [2009] to describe the water balance of temporarily inundated wetland sites dominated by stochastic rainfall.

2. Probability Distribution of Water Table Depth

[5] We consider an upward-oriented vertical axis with origin at the ground surface; the water table position is identified by $\tilde{y}$, negative below the soil surface, and the separation between saturated and unsaturated soil occurs at depth y, which corresponds to the top of the (saturated) capillary fringe. At this "saturation" depth, y, the pressure equals the (negative) saturated soil matric potential, ψ_s (also known as the air entry tension); above y, a one-to-one relationship between soil matric potential and soil water content is assumed. The vertical pressure distribution below the saturation depth y is taken as hydrostatic, thus the water table position below ground is $\tilde{y} = y + \psi_s$, while above ground $\tilde{y} = y$.

[6] The equation describing the dynamics of the saturation depth, y, in a homogeneous soil column at the plot scale reads

$$\beta(y)\frac{\mathrm{d}y}{\mathrm{d}t} = Re(y,t) - ET(y) + f_l(y), \qquad (1)$$

where $\beta(y)$ is the specific yield, $Re(y,t)$ is the groundwater recharge driven by precipitation, $ET(y)$ is the evapo-

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transpiration rate and $f_l(y)$ is the saturated lateral flow. Each term is defined piecewise according to the instantaneous position of the saturation depth, y, with respect to the soil surface and a critical depth, y_c, representing the separation between shallow (SWT) and deep (DWT) water table conditions. For the full mathematical description of the below-ground portion of each term, the reader is referred to *Laio et al.* [2009] and *Tamea et al.* [2009]; hereafter the complete form of each term is presented, including the case of water table above ground.

[7] The groundwater recharge rate is the result of precipitation, interception, infiltration and redistribution processes and represents the stochastic forcing of the dynamical system. The net rainfall reaching the soil after canopy interception is assumed to be a marked stationary Poisson process with mean arrival rate λ_0 and independent exponentially distributed depths with mean α. In SWT and waterlogged conditions, all rainfall events reaching the soil surface generate a fluctuation of the water table, whose amplitude is modulated by the specific yield, $\beta(y)$. Assuming that water infiltration and redistribution within the soil occur instantaneously, at the daily time scale, the recharge process is identical to the stochastic rainfall process. In DWT conditions, not all rainfall events reach the water table as the drier soil at the top of the soil column acts as a buffer on the stochastic process leading to a state-dependent recharge rate. The complete form of the recharge rate thus reads

$$\lambda(y) = \begin{cases} \lambda_0 \cdot \exp\left\{\dfrac{nh(y)\left[s_{fc} - \bar{s}'_m(h(y))\right]}{\alpha}\right\} & \text{if } y < y_c, \\ \lambda_0 & \text{if } y \geq y_c, \end{cases} \tag{2}$$

where $h(y)$ is the depth of the soil layer with soil moisture at field capacity and $\bar{s}'_m(h(y))$ is the long-term soil moisture average over the depth $[0, h(y)]$. The above equation is detailed by *Tamea et al.* [2009, equation 19].

[8] Evapotranspiration includes both plant transpiration and direct evaporation from the water table (above or below ground) and it is here modeled with a simplified approach. In bare soil without vegetation, the evaporation rate from a water table above ground is not very different from the evaporation rate from wet soil, if the water table is sufficiently shallow. Such a rate equals the atmospheric evaporativity, or maximum potential evapotranspiration rate, ET_p, which is imposed by atmospheric conditions such as air temperature and humidity [*Hillel*, 1998]. The same occurs in the case with vegetation, and the maximum potential rate is reached (i) in the case of shallow water table, mainly through plant transpiration and a small contribution of direct evaporation from water table, and (ii) in the case of waterlogging, where anoxic stress causes a reduction of plant root uptake, mainly through direct evaporation. Accordingly, in SWT and waterlogged conditions, the overall evapotranspiration is simply ET_p, independent on the water table depth and constant in time (considering long-term average atmospheric and climatic conditions). In DWT conditions, the direct evaporation is supposed to be negligible, and transpiration is reduced by plant water stress due to the lower soil moisture in the shallower soil layers. In these conditions, the evapotranspiration is expressed as by *Laio et al.* [2009], i.e.,

$$ET(y) = \begin{cases} ET_p\left(e^{\frac{h(y)}{b}} - e^{\frac{y}{b}}\right) & \text{if } y < y_c, \\ ET_p & \text{if } y \geq y_c, \end{cases} \tag{3}$$

where b is the mean depth of the vertical root distribution assumed as exponential.

[9] Finally, the groundwater flow, $f_l(y)$, quantifies the saturated lateral flow to/from an external water body with constant head, y_0. The flux depends on the difference in hydraulic head between the local water table ($\tilde{y}$) and external water body, and flux direction is either incoming or outgoing, depending on the relative elevation of the two free surfaces. Approximating the flux with a linear relationship, one can write

$$f_l(y) = \begin{cases} k_l(y_0 - y - \psi_s) & \text{if } y < 0, \\ k_{l,2}(y_0 - y) & \text{if } y \geq 0, \end{cases} \tag{4}$$

where k_l and $k_{l,2}$ are proportionality factors with $k_{l,2} \geq k_l$, in order to account for the possible surface runoff and direct connection of the two water levels when $y > 0$.

[10] The specific yield, $\beta(y)$, converts the volumetric variations of water stored in soil into water table fluctuations. $\beta(y)$ is a function of soil properties and soil moisture profile in the unsaturated zone, whose mathematical formulation is given by *Laio et al.* [2009, equations (20) and (30)]. Obviously, for a water table above the soil surface ($y \geq 0$), $\beta(y) = 1$.

[11] The long term probability density function (pdf) of y is computed from the stochastic differential equation (1) and reads [*Laio et al.*, 2009]

$$p_Y(y) = \frac{C \cdot \beta(y)}{ET(y) - f_l(y)} \exp\left\{-\int_0^y \left[\frac{\beta(u)}{\alpha} - \frac{\lambda_0 \beta(u)}{ET(u) - f_l(u)}\right] \mathrm{d}u\right\}, \tag{5}$$

where C is the normalizing factor reducing the pdf to a unitary area. The pdf of y has a lower bound, $y_{\lim}$, which corresponds to the equilibrium between lateral inflow and plant water uptake in the absence of rain. Above ground, equation (5) can be solved analytically thanks to the simple form taken by the terms inside the integral; such simplified expression, appropriately normalized to have unit area, represents the conditional pdf of above-ground water table depth and has the form of a Pearson type-III distribution. It reads

$$p_Y(y|y \geq 0) = C_2 \cdot (F + y/\alpha)^{\lambda_0/k_{l,2} - 1} e^{-(F + y/\alpha)}, \tag{6}$$

where $F = [(ET_p - y_0 k_{l,2}]/(\alpha k_{l,2})$ is a coefficient dependent on model parameters and C_2 is the normalization factor for the conditional pdf. Notice that below ground, the pdf of the water table depth, $p_{\tilde{Y}}(\tilde{y})$, is shifted of a negative distance ψ_s from $p_Y(y)$.

[12] Figure 1a shows an example of the pdf of the saturation depth, y, and the water table depth, $\tilde{y}$. Above ground the two curves coincide, while below ground they are offset by ψ_s due to the presence of the saturated capillary fringe. Notice that both $\tilde{y} = 0$ and $\tilde{y} = \psi_s$ correspond to a fully

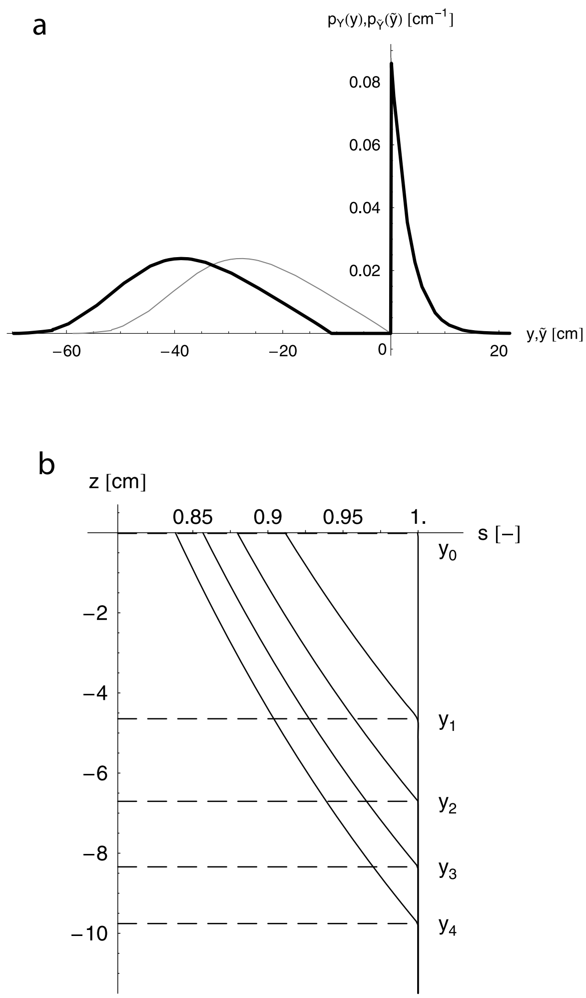

Figure 1. (a) Example of the probability distributions of saturation depth, y (thin line), and water table depth, $\tilde{y}$ (bold line), of a generic case study; the model parameters are detailed in Table 1. (b) Sequence of steady-state soil moisture profiles and saturation depths obtained by subtracting recursively ΔW = 1 mm of water, starting from complete saturation of the soil column.

saturated soil column (and soil surface), but while the former identifies a soil surface at atmospheric pressure, the latter has a soil surface at negative pressure, due to capillarity. Therefore, a rising water table is accompanied by a pressure jump at the soil surface from ψ_s to zero without water volume change; such behavior has been observed in field investigations and has been referred to as "Wieringermeer effect" [see *Heliotis and DeWitt*, 1987].

[13] Focusing on the pdf of the saturation depth, $p_Y(y)$, one can see a discontinuity at the interface between the porous medium ($y < 0$) and the air ($y > 0$). The extremes of such discontinuity are: *from the right*, the value dictated by equation (6) evaluated in $y = 0$, i.e. $C/(ET_p - f_l(0))$, and *from the left*, the zero value. The discontinuity results from the different water table dynamics above and below ground: above ground the addition/extraction of a water volume produces an equal variation of water table depth (β = 1). Below ground, the water table fluctuations are amplified by the capillarity in the porous medium, especially near the surface, where a small variation of water volume in the almost-saturated soil, produces a large variation in the water table position. This fact leads to an instability of the water table at the soil surface, which explains the left limit equal to 0, as detailed hereafter.

[14] Starting from a fully saturated soil column ($y = 0$), the initial extraction of a small water volume produces a disproportionate and sharp drop in the saturation depth. The portion of unsaturated soil has a steady-state soil moisture vertical profile that can be expressed by [*Tamea et al.*, 2009],

$$s(z, y) = \left[1 + \left(s_{fc}^{-1/2m} - 1\right)\left(\frac{y - z}{y_c}\right)\right]^{-2m}, \tag{7}$$

where z is the generic (negative) depth from soil surface, and m is the pore size index of the Brooks-Corey retention model [*Brooks and Corey*, 1964], while the water volume extracted reads $\Delta W = n \int_y^0 (1 - s(z, y))\, dz$. Subsequent extractions of the same water volume result in smaller and more proportionate variations of the saturation depth (see Figure 1b). The disproportionate reaction to the initial subtraction occurs because, although moisture in the unsaturated zone remains very high, the saturation depth immediately moves to deeper soil layers. As a consequence, the condition of complete soil saturation without waterlogging is unstable, and has a null probability to occur.

3. Results and Discussion

[15] The stochastic model for water table dynamics is validated with water level data in the Everglades National Park, which are collected and made publicly available by U.S. Geological Survey at http://water.usgs.gov. Records have been chosen for the time series length, the water level fluctuations occurring both above and below ground, and the availability of rainfall/vegetation/soil information from the same area. The analysis focused on groundwater wells NP46 and NP67 and considered the growing seasons, i.e., from May 1st to October 31st, in the period from 01/01/2000 to 06/07/2009. The model parameters used in end case and detailed hereafter are summarized in Table 1.

[16] Rainfall data are based on the Next Generation Radar (NEXRAD) data and are made publicly available by EDEN at http://sofia.usgs.gov/eden; at the groundwater well sites, the reconstructed time series range from 01/01/2002 to 31/12/2008. Rainfall records cover only a portion of the time interval considered but provide an accurate estimate of the local precipitation pattern, which is quantified with

Table 1. Parameter Values Used in the Case Studies

	λ_0 (1/d)	α (cm)	ET_p (cm/d)	b (cm)	y_0 (cm)	k_l (1/d)	$k_{l,2}$ (1/d)	soil	n (-)	ψ_s (cm)	m (-)	k_s (cm/d)
Example	0.30	1.2	0.4	20	−20	0.007	0.007	loam	0.463	−11	0.22	31
Everglades-NP46	0.589	0.95	0.48	10	0	0.001	0.006	marly-peat	0.5	−10	0.20	10
Everglades-NP62	0.607	0.97	0.46	10	−20	0.001	0.003	marly-peat	0.5	−10	0.20	10

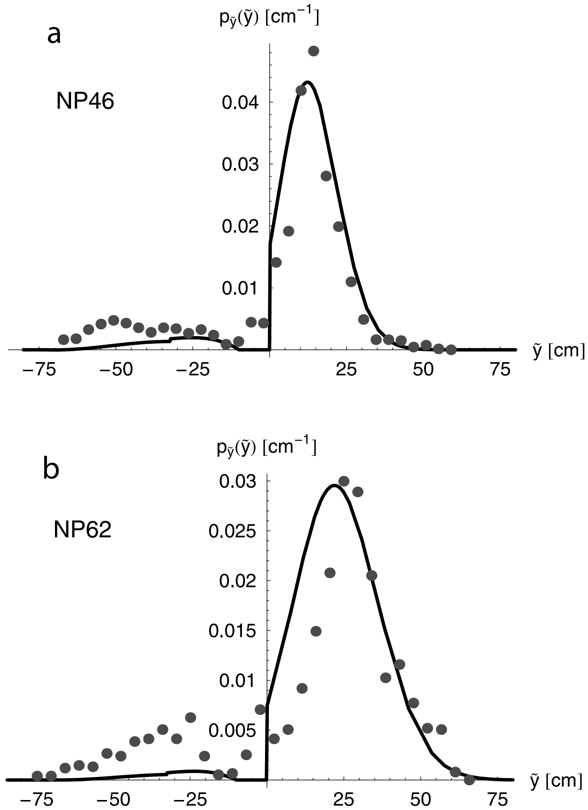

Figure 2. Comparison between model pdf and normalized histogram of data in two sites of the Everglades National Park: (a) NP46 and (b) NP62; data correspond to the growing seasons and model parameters are summarized in Table 1.

the average rainfall depth, α, and average arrival rate, λ_0, in the growing season.

[17] Evapotranspiration information is provided by *EDEN* and the potential evapotranspiration rate, ET_p, has been estimated as the average rate over the growing seasons. Vegetation mainly includes wetland shrubs and emergent marsh with shallow roots laying in the top 20 cm of soil [*Yates*, 1974]. Assuming an exponential root vertical distribution, we estimated the mean root depth, b, to be about 10 cm. The soil at the two sites is wetland marly-peat, as indicated by the *FCE-LTER* website; the soil hydraulic parameters, n, m, ψ_s and k_s, have been estimated considering characteristic values for such parameters, on the basis of the extensive analysis performed by *Myers* [1999].

[18] The external water body is the sea, located at a distance of 10 to 20 miles; the depth y_0 is taken as the mean sea level relative to the site elevation (that is, using the site elevation as the datum). The lateral flow coefficient, k_l, is estimated on the basis of groundwater data: the deepest water table position reached during the monitored period is taken as an estimate of $y_{\lim}$. Under this assumption, k_l can be derived from equation (4) evaluated at $y_{\lim}$, where lateral flow is in equilibrium with evapotranspiration, i.e.,

$$k_l = \frac{ET(y_{lim})}{y_0 - y_{lim} - \psi_s}. \tag{8}$$

The lateral flow coefficient for water levels above ground, $k_{l,2}$, is taken as a tuning parameter. The values of k_l and $k_{l,2}$ are similar if there is limited hydraulic connectivity between local standing water and external water body, for example due to a large distance of the latter or to local topography, including hollows and ridges.

[19] Figure 2 compares the probability density function (pdf) predicted by the model to the frequency histogram of water level data. Good agreement between the two pdf's is observed, and especially the ranges above-ground are well captured, both in position and shape. The variability of water table position below ground is more difficult to reproduce; in fact, the model partially underestimates the probabilities in this range although it correctly captures the bimodal character observed in the empirical pattern. The reasons for such underestimation can be several: soil inhomogeneity, local topography, temporal variability of the parameters (e.g., y_0 or rainfall), to name a few. Nonetheless, the model is capable to describe reasonably well the stochastic dynamics of water depth with a limited number of parameters, most of which are evaluated from direct observation.

[20] A final comment is necessary about the discontinuity of the water table pdf at the soil surface. Sometimes groundwater data do not show the marked variation predicted by the model and physically justified above; such fact can be explained as follows. On the one hand, the model describes the soil water balance in a flat-top, uniform and undisturbed soil column, which are conditions sometimes different from field, on the other hand, the presence of local water redistribution can hide or soften the sudden pressure drop and the water level change, when the groundwater goes below ground. Micro-topography at the soil surface and non-homogeneity in the top layer of soil, as well as the presence of the piezometric well itself, generate local water redistribution and affect groundwater fluctuations near the soil surface. Possible mechanisms occurring when the water table drops below the ground surface can be of two types: i) the local accumulation of water in hollows or macropores, which is released when the free surface inside the piezometer drops, fills the gauging well and partially compensates the sudden change in the free surface position, or ii) the inertia of the piezometer, the piezometer responds to the water table drop by slowly releasing the excess of water to the surrounding soil and increasing the local soil moisture. In both cases, the water table dynamics at the interface above/below ground is modified by the presence of the groundwater well itself, which disturbs any groundwater measurement collected in this way. It is worth noting that water level measurements performed with buried pressure transducer are more accurate and can capture the pressure jump occurring near the soil surface [see, e.g., *Turner and Nielsen*, 1997].

[21] In conclusion, this paper introduces a stochastic model for water level fluctuation in wetlands sites which are temporarily inundated. Reasonable assumptions and few parameters simplify the modeling approach and provide a convenient tool for studying wetland ecosystem dynamics, which performs well when compared to the empirical data. Some limitations stem from the inherent simplifications and the large uncertainty associated with quantitative measurements of soil water, as well as to site-specific hydrological and physical parameters. However, achieving the goal of capturing the water table stochastic variability, the model is suitable for investigating several important problems related

to ecohydrology, plant dynamics, and biogeochemical processes in wetland ecosystems, and for establishing a quantitative understanding of these important ecosystems.

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